Archie's Rocks in Virtual Laboratory

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Introduction

This paper demonstrates successful development of Archie's formula with rock samples that were created and tested in our virtual rock laboratory. The calculated values of 'm' and 'n' exponents are in range with estimates for the real rock samples. The concepts, simulated processes, and diagnostics supported by the Virtual Petrophysical Laboratory (VPL) were presented in the first three papers[1,2,3] during Geoconvention in 2020, 2021, and 2022. The last paper presented during Global Petroleum Show, Calgary, June 2022 outlined the most advanced processes based on dynamic changes in pore space and changes to the pore matrix[4]. Developing new concepts and processes with computers required extensive testing and validation. This validation study will lead to improvements in processes and algorithms.

Data and Rock Models

At the beginning, 20 rock samples were built with different pore and grain size distribution probabilities using an octree data structure. 13 samples had homogeneous structures while remaining seven had a correlated pore network with higher pore probability in the horizontal direction. In addition, we built and tested a 3-layer model (including nonconductive zone) with a correlated pore network and a small effective porosity of 5%, which was strongly conductive. Finally, we tested a uniform sample with extremely low effective porosity of 1.8%, which had a break through events during simulations of the drainage and imbibition tests.
The majority of models were equivalent to 134 million equal size cells that were compacted into less than 35 million cells of variable size.
All 20-sample data in our list was used to build a general formula. In addition, we created separate formulas for the homogeneous samples (uniform) and samples with horizontally correlated pore networks. Equations of the resistivity and the formation factor versus the effective porosity were created for comparing different upscaling algorithms. These equations estimate Archie's 'm' and 'n' exponents.

Synthetic Samples and Their Characteristics

A wide range of pore and grain size distributions were used to create synthetic rock samples with very different properties. The porosity of models varied from around 20% to 33%, while their effective porosity (interconnecting two opposite model faces) varied from 2.5% to 31%. Samples with homogenous pore structures had similar properties in horizontal and vertical directions and had low conductivity at lower porosity. Seven samples with correlated pore networks conducted fluid, even when their effective porosities were very low (e.g. 2.5% and 5%). We define the effective porosity as the portion of a pore network that maintains communication between two opposite sides of a sample.
In simulations following initial steps a sample is fully saturated with a brine. Next, electrical resistivity is estimated with two different methods: using statistical averaging and using electrical

equivalent circuits. In the second case, electrical circuit configurations adjust with the flow direction. Thus, the horizontal and vertical estimates may be different for the non-uniform rock samples. Resistivity of octree octants is upscaled in a bottom-up tree recursive traversal. Details were provided in our last paper [4].

In our most advanced mesh model that was designed for this project, voxels are replaced by sets of six resistors, organized as 3D crosses. This approach is similar to simulations of porous media with a regular cubic lattice corresponding to a network coordination number of six. In this solution an extended local network is built from a set of eight voxels (siblings) which have the same parent and they are replaced by 48 electrical circuit elements.

This schema allows cells to have different properties in horizontal and vertical directions. The nodal analysis is applied to this circuit to solve for voltages at electrical nodes which correspond to connections between resistors of the network[4]. In our solution solid matrix properties are included in computations based on their properties, which is not too common.

Results Analysis and Interpretation

A wide range of pore and grain size distributions were used to create synthetic rock samples with very different properties. Initial data analysis and results showed drastic differences between upscaled resistivity values depending on the pore network and the upscaling method.

Analyses have shown that arithmetic and geometric resistivity averages are often too high. This would lead to misclassifications of good samples into non-conductive samples. The harmonic averages seem more reasonable but their horizontal and vertical values are equal in both directions, while our algorithms based on electrical circuits account for heterogeneity and estimates are done for each flow direction. The mesh model has values that fall between the other two models with values similar to the chain model but better regression fit for log-log lines of the resistivity as a function of the porosity. Thus, in the rest of experiments we used the mesh model and developed relationships between the formation factor and effective porosity as shown in the next section.

Archie's Equation

Our validation is based on the original Archie equation (1).

$$S_w{}^n = \frac{R_w}{(\Phi^m \times R_t)} \quad (1)$$

where
Rw=water resistivity
Rt=rock sample resistivity
Sw=water saturation
m=cementation factor
n=saturation exponent
Φ=porosity

For a fully saturated sample we define Rt/Rw as the formation factor (FF) and we fit:

$$FF = \Phi^{-m} \qquad (2)$$

Our final formula has an extra coefficient when fitting a power curve. Figure 1a presents overall FF model characterized by a relatively good fit R^2=0.78. Much stronger Archie's models were built for each rock type (uniform versus correlated pore network) with both R^2 > 0.80 (Figure 1b).

The overall 'm' exponent is 2.34. The uniform samples' exponent 'm' is 2.26 while the correlated network samples have lower number equal to 1.92 (see Table 1). The developed equations include a formation dependent constant C. It is often referred to in papers as a tortuosity factor[5].

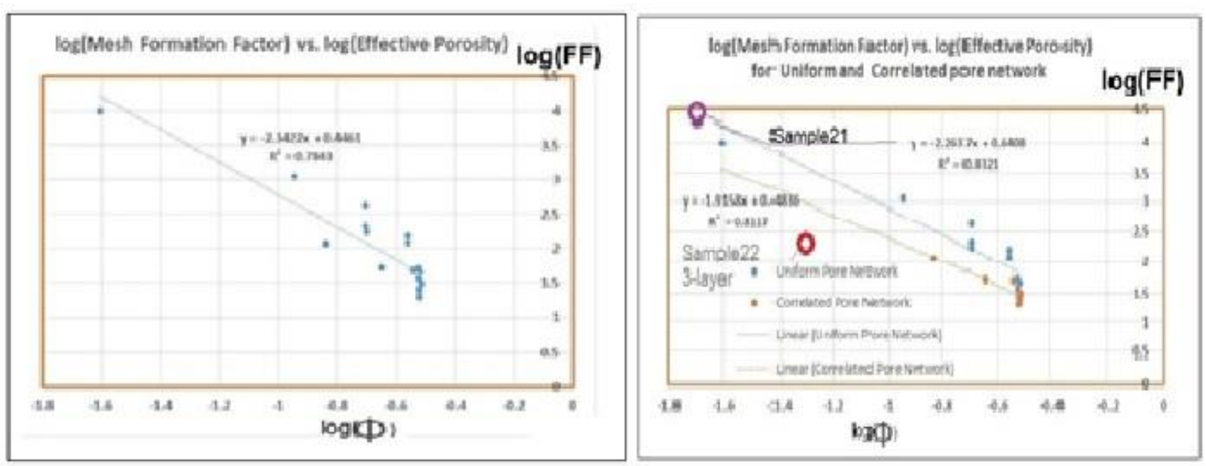

Figure 1. a) Overall Mesh FF model (all 20 samples); b): Mesh FF by rock type.

Table 1. Archie's parameters.

Model	m	C
20 samples	*2.34*	*2.79*
Uniform	2.26	4.37
Correlated	1.92	3.05

In addition, a large red circle below both lines projects the 3-layer Sample 22 with correlated total porosity of 19.9% and the effective porosity of 5%. This layered model has the smallest FF=218 of all samples with the same porosity. The highly conductive layer acts like a short circuit in electrical systems.

The uniform samples have higher resistivity than corresponding samples with correlated pore networks in the specific direction.

Sample 21 has 20% porosity and 1.8% effective porosity, and the highest FF=28k. This sample corresponds to the Sample 20 with similar 21.3% porosity and 14.6% effective porosity, and a much smaller formation factor of 670. This sample fits the derived Archie's formula for the uniform pore networks as shown in this Figure 1b.

Thus, Archie formula is still valid for samples with low porosity created in our system. Models built here can be multimodal, contain different fluids, and can have conductive matrix elements without restrictions reported in other studies of Archie's pore networks[5].

Results Presentation and Visualization

Figure 2 presents visualization and testing of a sample using an octree with eight levels of subdivision. The model was represented by a low-resolution octree with the total nodes equal to 4,960,673 and the total-leaf-nodes equal to 4,340,414. This shows how the adaptive data structure of variable size cubes can help in upscaling by compressing homogeneous spaces into larger units. Higher levels of compression can be achieved with higher number of the subdivision layers. The original sample had a correlated pore network and participated in developing the original Archie's equations in earlier sections. Figure 2 compares capillary (saturation) pressure curves of this new homogeneous sample with curves of the original sample with the correlated pore network.

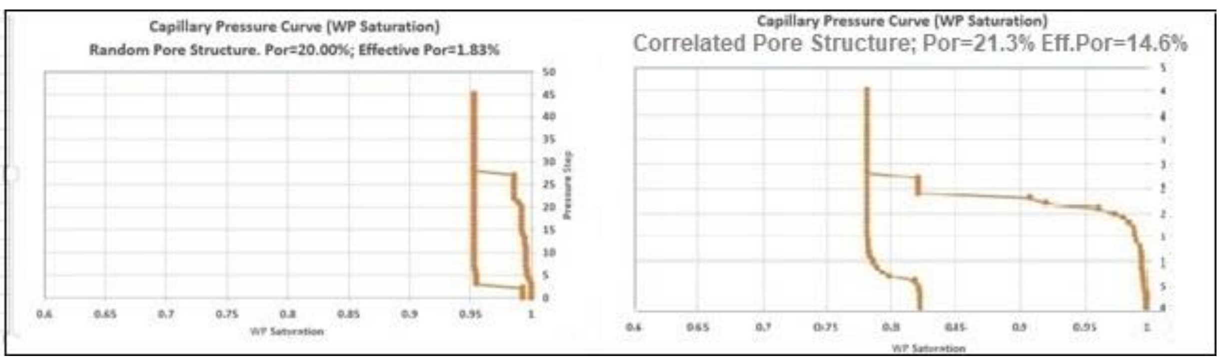

Figure 2. Wetting phase saturation versus pressure for the uniform (random) Sample 21 in comparison to a similar Sample20 with a correlated pore structure.

Specifically, the total porosity of the homogeneous sample was 20.0% and the effective porosity of 1.83% while the correlated sample had 21.3% and 14.6% respectively. Thus, the homogeneous sample had very little effective porosity, however it had a breakthrough when simulating capillary pressure curves (Figure 2). The right diagram (Figure 2) shows much larger water saturation changes during the simulated test. Figure 3 shows changes in the capillary pressure curves developed with resistivity estimates in the horizontal and vertical directions. The upper graphs (representing horizontal and vertical directions in Figure 3) have very narrow hysteresis curves with high resistivity.

Finally, our visualization system supports the display of selected elements of the pore network. This allows visualizing the different stages in drainage and imbibition at specific pressure points. For example, Figure 4 presents only an active part of the pore network (Sample 21) at the breakthrough during the early stage of the drainage simulation. Thus, it shows only cells on-interface of two fluids.

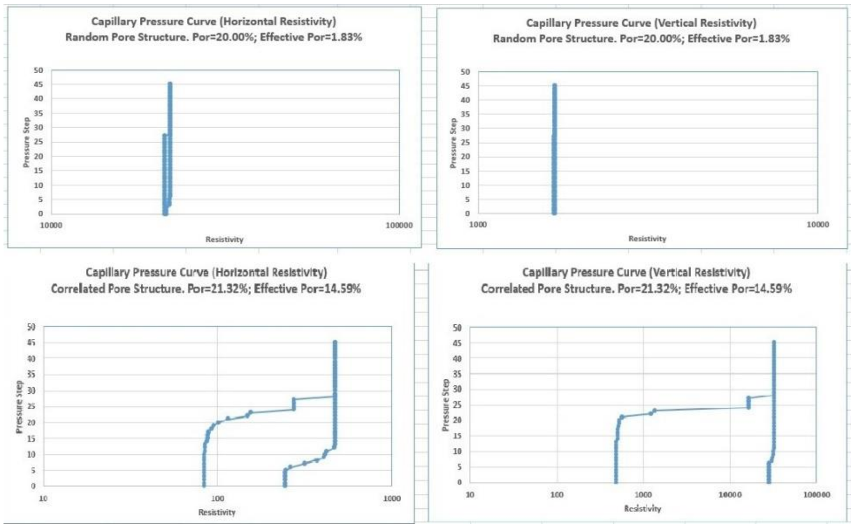

Figure 3. Resistivity versus pressure for a random Sample 21 in comparison to Sample 20 with the correlated pore structure.

Figure 4. Sample 21 at the breakthrough during drainage; Porosity=20%; Effective Porosity =1.8%; Non-wetting phase is Blue and Wetting phase is Green.

Resistivity Index and Water Saturation

We estimated the formation resistivity index and the wetting phase saturation in a series of pair-wise values obtained during the simulation of capillary presume curves. The slope of these log-log lines gave us the saturation exponents for each sample. It was done separately for the drain and imbibition curves. The majority of samples have a good fit. Figure 5 presents saturation plots for Sample 5 with an extremely good fit across of the whole range of the drainage and imbibition test runs.

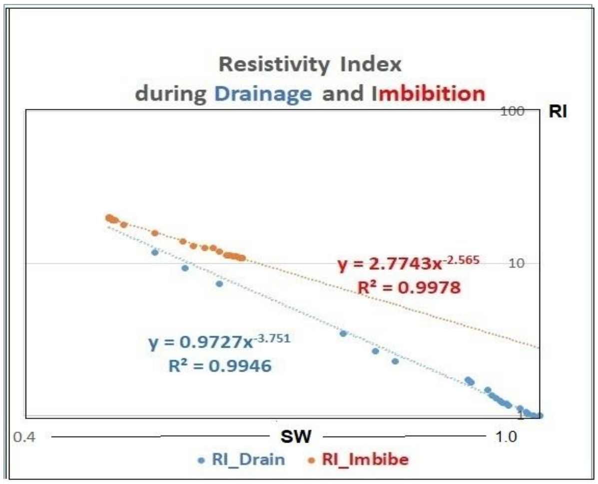

Figure 5. RI=f(SW) curve for Sample 5.

Sample 5 was introduced in the first part of this study and it had total correlated porosity of 30.6% and effective porosity of 28.7%. As shown in Figure 5 the drainage saturation exponent was n=3.7 and the imbibition exponent was n=2.8. The fitted lines corresponding to these evaluations had R^2 well above 0.99.

The drain coefficient=0.97 is close to 1.0 what corresponds well to the starting point of the 100% wetting phase saturation. The imbibition corresponding numbers for the same sample were: R^2=0.99; n= 12.79;

Literature search confirmed that the standard n=2.0 is not expected for samples with complex pore geometry, high wettability, and high-water saturation. The expected range of n can be higher than 4 and can reach 20 plus[8]. The majority of samples in our study had n between 5 and 8. However, we had samples with values between 10 and 20, while the imbibition part had some exponents higher than 20.

Research Novelty

Our VRL prototype allows us to build synthetic samples, run simulations, and derive relationships and formulas that are used in the petroleum industry. The system uses only cubes of different size to build rock samples without any intermediate representations. Thus, all parts of the pore network including grains, pores and fluids, are built using the same elements (cubes of different size but with different properties). Grains and other solid or additional liquid elements participate in modeling and parameter estimations. A typical Archie rock[5] is water-wet and contains brine as the only conductive phase. It comprises nonconductive, approximately equidimensional grains with a simple unimodal intergranular pore system.

In more realistic solutions, samples are sorted out into ‘rock types’ and/or permeability classes[6,7]. Our model does not need these rock types or classes. Instead, each cell has a ‘known’ property and it does not have to be represented by simplified geometric concepts. The matrix, cementation, intrusions, and interacting fluids are still cubes and they all are treated the same way in recursive algorithms.

This study shows that statistical upscaling and calculations based on intermediate elements (pipes, bundles, or resistor combinations…) bring uncertainty and require calibration and verifications that influence results. The octree-based upscaling of the sample resistivity captures differences between estimates in the horizontal and vertical directions. Rock matrix properties are included in the estimates and there is no limit on the rock or fluid type that are to be included in the processes and estimates.
Archie’s equations can be developed and tested with all of the data or selected segments of the population. These equations explain a sample’s behavior in extreme cases without additional sample segmentation, and rock classifications. These formulas confirmed that our upscale mesh algorithm works the best.

Summary and Future Research

Here we present a software prototype, which builds a range of rock samples and simulates processes significant for the petroleum and other industries. The designed software and processes create and test large pore network structures directly in 3D space without intermediate representations or using different elements to represent porosity, micro-porosity, rock matrix, etc.

The majority of models developed and tested during this project exhibit the classical Archie behavior and consistent m and n exponents in a variety of complicated pore networks. Archie’s formula derived from this study is similar to reported based on actual laboratory experiments. The estimated overall Archie’s m exponent is 2.34, while the sample with the uniform pore network has m=2.26 and homogeneous group has m=1.92. A typical n exponent estimates are between 5 and 8 with some above 10. Furthermore, the imbibition n values are typically higher than their corresponding drainage exponents.
This study shows that the electrical resistivity and formation factor estimates vary from probable to completely erroneous when applying poorly tested upscaling algorithms. Our mesh upscaling algorithm performed well in all porosity and saturation ranges.
At the same time we have proven that VRL enables testing scientific hypotheses and helps in the understanding of "cause and effect" in porous media.

Finally, we observed that some experiments and estimates will benefit if done with higher than 1024*1024*1024 max resolution used in this study. The next step with the cube of 1/2048 is required to adequately account for micro porosity.

Over the years, tortuosity has been given a lot of attention in many papers[7]. Specifically, how it is related to a product of the formation factor and the effective porosity. In VPL many paths are measured when traversing the octree. These values and associated statistics will be included in our next paper.

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